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ON THE SUPREMACY OF VISCOSITY  
IN THE CONTROL OF TURBULENT  
FLUID MOTION

MAX M. MUNK

**FC**

ON THE SUPREMACY OF VISCOSITY  
IN THE CONTROL OF TURBULENT  
FLUID MOTION

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The Catholic University of America

April 1956

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On the Supremacy of Viscosity in the Control  
of Turbulent Fluid Motion.

Summary:

Even for Reynolds numbers not small, the dynamic effects including the dynamic velocity terms and the pressure terms enumerated in the Navier Stokes equation exert small average effects on the turbulence features when compared with the effects of viscosity.

This new finding points to the linearization of the turbulence problem. Non-linear effects of the statistical deviations from the random distribution, however, are not insignificant.

The theory is applied to the determination of the ratio of the two intensities of turbulence mixing, the one being the shear mixing and the other the energy diffusion mixing.

## I. Introduction.

The turbulent fluid motion presents a problem in statistics. It must be the aim of any real turbulence theory to sift experimental and theoretical knowledge to the utmost, so that we may be able to predict a small number of relevant averages by the use of as small a number of averaged variables as possible.

We have shown in Ref. (m) that even the restriction to three such variables affords a good insight into the general features of turbulent fluid motion. This encourages us to go forward in the chosen direction. The present paper deals with refinements of the mathematical procedure. It brings our approach in line with the branch of the current literature generally summed up as "statistical theory of turbulence." This is not difficult in principle because an average lump flow is not quite unrelated to the components of the velocity correlation dyadic. The lump picture and the abstract correlation view are in harmony with each other. The correlations appeal more to the abstract mathematician. The lumps seem to offer more hope of progress to the physicist who is aware of the danger of pitfalls of pure abstract reasoning.

Time and space forbids us to present a self-contained account of the current statistical approach. The following is accordingly directed to readers who are reasonably acquainted with at least the two pioneer papers on the subject, by G.I. Taylor and Theodore von Karman. Ref. (t) and (k). A relatively recent account by Agostini may also be consulted with profit. It gives a good idea about the

present status of the theory, about its satisfactory aspects as well as its weak ones. An excellent translation of Agostini's paper has been made by J. Vanier, and is readily available. Ref. (a).

II. The postulates of the statistical theory.

All postulates employed in the statistical theory of turbulence can be classified under the heading of three different principles. They flow either from the physical theory, or from an attempt to separate a turbulence motion from a basic motion, or lastly from some kind of artificial specialization.

The physical theory by itself is not uncertain nor controversial. It is exhaustively laid down in the Navier Stokes equations in combination with the equation of continuity. It is implied that the solutions depend also and in a most pronounced way on the boundary conditions clearly necessary or merely assumed. The Navier Stokes equations are ordinarily stated with reference to incompressible fluids only. When adopted in that special form, it is out of place to speculate again whether the fluid is assumed to be incompressible or not.

The division of the actual flow into a basic<sup>smooth</sup> flow and a turbulence flow is quite generally resorted to. But it is not always realized that this division is not unique. It will not do just to say that the basic flow is the average, and the turbulence flow is what remains over and above the average. That is not precise enough. The division has also to include the boundary conditions. Also the averaging process can be carried out in more than one manner and to different degree and intensity. There is in fact no sharp demarcation line between the turbulence flow and the basic flow. If we



carry the averaging too far all motion becomes turbulence motion. If not far enough, the basic flow remains too involved.

Specialization, such as to homogeneous flows and to isotropic turbulence, is resorted to for dividing the difficulties of the problem. It is attempted to investigate in that fashion different aspects of turbulence by themselves. This is done with the expectation that later these different effects may be combined or superposed to each other in some simple way. This subdivision and the postulates necessary to effect it are again not unique nor precise, but they are particularly tricky and readily lead to gaps in the logical procedure.

An orderly progress in the presently discussed theory requires that the postulates necessary for applying these three principles be introduced gradually and with greatest care. The decisions should be delayed until they become immediately necessary in each individual case. That particular case serves then for illustrating and explaining the scope of the postulate. It leads only to confusion if some so-called definition is accepted as a dogma to serve as the basis of the consequent research. The mathematical procedure includes then "proving" that certain relations hold by trying to demonstrate that they conform with the perhaps ambiguously stated or prematurely adopted definitions.

It may seem strange that such reserved attitude should also be necessary with regard to the physical theory, which stands established certain and rigid. But the boundary conditions are a matter of discretion, and cannot be established without regard to physics. Navier Stokes equations do not determine the boundary conditions.

Said equations are precise when applied to specific solutions. But there is no solution to apply them to. It is the very nature of the turbulence problem that the precise solution must forever remain unknown and unstated. The application of the equations in such cases requires mental steps not apparent from the equations. The formulated equations stand for broad principles, not for a narrow numerical procedure. A full grasp of all implications of the equations has to be constantly resorted to.

All applied statistical theory is attended with some lack of rigour. Thus for instance, the standard or Gaussian distribution is often used even if it is not the true distribution. The theory deals with incomplete, scant and uncertain data, and conclusions regarding a shape have, so to speak, to be drawn from its vague shadow. Under such circumstances we cannot afford over-fine mathematical distinctions to stand in the way of using quite workable and practical propositions, unprovable as they may be. The door to ingenuous and shrewd guesses must not be closed. Simplifying assumptions should not be rejected for lack of mathematical rigours unless it is demonstrated that they lead to unacceptable conclusions. In such manner was the kinetic theory of gases developed. The turbulence problem is a much humbler problem, having no significance by itself but only having a significance on account of engineering needs. Certainly, what is good enough for fundamental physics is good enough for turbulence research.

### III. The correlation flow.

The correlation flow is obtained by a scalar multiplication of the correlation dyadic (tensor of second rank)

$$\bar{v} = i \cdot \overline{u; v} / \overline{u \cdot v}$$

R is the correlation dyadic  $u; v$ , where the semicolon has no significance beyond that of the empty space ordinarily employed for indicating tensor multiplication. The symbol "i" denotes a unit vector. An entire tensor field R is associated with each point of the space. Each field is in the associated space, and only v but not u varies in this space. All variables depend besides on time, all averages are time averages. The divergence is zero

$$\nabla \cdot \bar{v} = 0$$

because averaging leaves absence of divergence unchanged.

It is always postulated that the turbulence is reasonably homogeneous so that the changes of the averages through the territory of one lump are negligibly small when compared with the averages themselves. This postulate is often unrealistic.

The present discussion is restricted throughout to homogeneous and incompressible fluids. The correlation flow is the average of the turbulence motion at one point provided the average is only taken while the velocity at that point is reasonably near to one predetermined direction. That is certainly so for isotropic turbulence. We see or assume that the correlation flow possesses a number of symmetric features. It is ordinarily assumed to possess an axis of symmetry

parallel to the central velocity. This does not mean yet isotropic turbulence. We shall see or assume that the correlation flow possesses a number of symmetric features. It is ordinarily assumed to possess an axis of symmetry parallel to the central velocity. This does not mean yet isotropic turbulence. Only if the correlation flow is the same for all directions is the turbulence isotropic. We hope to be able to reach tangible results even in the case that the turbulence is only approximately and merely to some degree isotropic, by using suitable averages of the correlation flows in different directions.

The correlation flows reach theoretically up to the infinity. They converge rapidly to zero, as we shall discuss in fuller detail in Section 5. Still it is impossible to assign a sharp boundary up to which the flows actively reach. A similar situation was already faced with our Hill vortices in Ref. (m), and is quite common in applied physics.

There is a separate field or space function of correlation dyadics and therefore also one of correlation flows for each point. In that respect the correlation flow is different from the lump flow. Also lumps have a physical reality, but correlations are merely probability weight functions. They change with the change of the entire flow but they have no individual history, because they have no individual physical existence. Thus, in a quasi-steady turbulent flow the correlations and therefore the correlation flows do not change

The individual lumps change each by themselves even their average

never changes. The lump flow has always a history of change. It is the plan of our inquiry to study the average history of the individual lump flows as already done in Ref. (m).

The average lump flows are closely related to the core or inner portion of the correlation flow. The two are not strictly identical but they suggest each other. The lump flows, even the average lump flows, are a physical reality and it is not obvious that a certain correlation flow corresponds to one lump flow only. The study of such detail questions is however contrary to the plan of a statistical approach. We shall never know either of the two flows with any precision. The distinction does not appear relevant at this state of the inquiry.

We plan to disregard the distinctions between the core of the correlation flow and the lump flow, except for a proportionality factor for the velocity magnitudes. The strength of the lump flow in relation to the strength or energy density of the turbulence depends on the average lump distance. We plan to present an inquiry about that in a later paper. The present paper deals with the supremacy of the viscosity only.

#### IV. Correlation dynamics

In current literature on the statistical theory of turbulence the term "dynamics" designates the study of the time changes of the correlation dyadic of pairs of simultaneous velocities. If the flow is quasi-steady, there is not any such change. But the theory is mostly applied to isotropic turbulence. Then the turbulence always decays and the correlation dyadic changes.

The fluid has to rely on pressure effects for the preservation of density. The current literature notes that nevertheless, for isotropic turbulence, the pressure terms of the Navier Stokes equation cancel out by averaging, and leave no trace in the equation describing the history of the correlation. The correlation dyadic undergoes changes just as if there were no pressures acting. This proposition is ordinarily accepted on the authority of von Karman, Ref.(k). The proof or demonstration given there seems to be not generally accepted. Most authors do not discuss this point. Agostini, Ref. (a), page 69, discusses the disappearance of the scalar product of the velocity and the pressure gradient, not their dyadic product. Both products show equal response or non-response to the pressure. Agostini does not accept the proposition. A few pages later, Ref.(a) p. 75, he says: "When assuming the disappearance of the pressure term to have proved." In effect he repudiates with these words the proof he has himself offered and rejects the different proof of von Karman which he must have studied carefully. The present author, while realizing that Ref. (k) Section 7 is perhaps not quite as lucid as it could be in view of the simplicity and directness of the reasoning presented, still does not find any error or fault either in the

postulates employed nor in the rather primitive and elementary steps taken. The pressure terms do disappear. It is however noted that the demonstration in said Section 7 holds equally for any other scalar variable subject to the condition of statistical isotropy. Nothing is introduced in Section 7 having reference to the scalar  $w$  representing the pressure. This widens the scope of application of the result of Section 7 in that it applies to all isotropic scalar variables, but it also narrows it unnecessarily in that it restricts the disappearance of the correlation dyadic to a product of the velocity and of a vector belonging to an irrotational vector field.

The final outcome appears in quite a different light when relying more on physical grounds and less on mere geometry and postulates. The pressure terms are then seen to average out without having to rely on pressure gradients being derived from a scalar potential. They average out for a much broader reason, namely because the pressure is an even function of the velocity. If all turbulence velocities are permitted to be reversed, the pressure distribution would remain what it was before, without change of sign. It is therefore reasonable to assume that for each pressure value all velocities occur as often in one direction and magnitude as in the opposite direction and the same magnitude. This leads immediately to the disappearance of the pressure-velocity correlation gradient. Neither a potential of the pressure force, nor isotropy of the turbulence motion is necessary for proof. Now, this apparently new mode of reasoning

is of larger significance than just furnishing a new proof for an old and long accepted proposition. It widens the scope of application to less restricted turbulence and to other terms of the equation. It directs our attention to the remaining dynamic pressure terms of the Navier Stokes equation. They can be written

$\nabla \cdot (v; v)$  and they are also an even function of the velocity.

Exactly the same reasoning that holds for the pressure terms applies also to the dynamic velocity terms. These terms will also cancel out on the average for the history of the correlation dyadic and of correlation flow. That puts the theory in an entirely new light. Only the viscosity terms remain, governing the correlation dyadic and the correlation flow, even if the Reynolds number is not particularly small.

Retaining merely the viscosity term was already considered by von Karman, Ref. (k) Section 10, but expressly restricted to "small Reynolds numbers." We see now that the supremacy of the viscosity is quite general. Agostini, Ref. (a) Section 20, also discusses this discounting of the dynamic velocity terms, but with apology and little faith. He defends it by stating that it is mathematically convenient but physically quite difficult to justify. He overlooked the physical justification presented in the present section believing rather that there was none.

The correlation distribution is thus seen to be subject to a linear differential equation. This situation must however not be accepted without reservation. What happens in the turbulence



flow does not depend merely on the probability distribution but also on the deviations from that probability. Secondary deviation effects must not be overlooked. Section 7 of this paper illustrates this remark.

V. The correlation flow at large distances.

It is necessary to clarify the meaning of the total momentum integral, that is the space integral of the correlation velocity vector,

5-1

$$\int v \, dS$$

the integration extending through the entire space. Also that of the space integral for the total polar moment of inertia of the correlation velocity,

5-2

$$\int r^2 v \, dS$$

where  $v$  denotes the velocity,  $r$  denotes the radial distance from the center, and  $dS$  denotes the space elements.

With one Hill vortex or with a similar correlation flow the velocities decline as the cube of the radial distance  $r$ . Hence the momentum integral (5-1) does not converge. Its value depends on the order in which the integration is carried out, and on the specification for the infinitely far boundary. Integral (5-2) would even less converge.

We have to look deeper into this question of convergence. We must call to mind that we are facing not one single lump but an infinite assembly of lumps, moving in some random fashion about in the fluid. Their distribution is not entirely random in that there is a bias or correlation between the motion of adjacent lumps. This is

the case by reason of the physical laws governing what happened in the past. Individual lumps have only a short life span, and they are all created by the action of the same pressure distribution. The momentum of the fluid is not changed but merely redistributed. In consequence each lump traveling in one direction is surrounded by a group of lumps having on the average a total momentum in the opposite direction. Hence, the correlation spreads out far beyond the territory of the lump. Also this momentum is evenly distributed around the first mentioned lump. There is no upper limit for the statistical symmetry thus established. In consequence, the correlation velocity declines much faster than inverse to the third power of the distance. It would at least decline with the fifth power; but there is no upper limit of the exponent  $m$  by <sup>which</sup> the velocity may decline with the distance. An exponential law  $e^{-kx^2}$  seems appropriate and acceptable.

This must not be taken too literally. The sense of reality must be preserved. The higher the power, the slower the convergence, and the larger the deviation from the mean. It will require a longer and longer time to obtain the average and larger space too. The average space integrals become very unreal as the power  $m$  goes up. It is impossible to determine them experimentally. But for the following we do not have to go very high with the exponent  $m$ . In consequence of this physical reasoning we feel justified in postulating that all correlation integrals occurring converge at large distance from the origin.

VI. On the preservation of the lump flow.

Additional comment regarding the minor part played by the inertia effects in the turbulence theory seems to be called for. It is clear that each individual lump is attended by a pressure distribution and by inertia effects. We have already pointed out in Section 4 why these inertia effects are secondary with regard to average effects. We would like to show this in<sup>a</sup> more tangible way using the physical picture of one lump undergoing changes.

If a lump, under the action of inertia only, preserves its compactness, the flow picture will, to some extent, be more or less periodical, and therefore its time average will be constant. The basic question is accordingly whether a compact lump flow, of the type considered, which is a generalized vortex ring in a non-viscous fluid similar to the Hill vortex of Ref. (m) remains compact. The opposite would be that the vortices spread out to become distributed through a larger and larger territory, i.e. in the absence of viscosity. The vortex rings of the correlation flow occur equally in both directions but the vortex rings of the lumps are predominantly in one direction. The vorticity distribution through the outer territory of the correlation flow can not count for the lump, because that consists of many lumps rather than of an even and gradual distribution of vorticity. Now in absence of viscosity effects, the total energy of the lump flow must remain constant. The total kinetic energy of the flow depends on the compactness of the vortex distribution. The wider the same vortex lines are scattered and distributed, the smaller is the kinetic energy of the flow. Ref (1) Section 153. It follows therefore from the

energy principle together with the fact that predominately only vortex rings of the same direction are involved, that the lump flow must remain compact. It is therefore statistically self-preserving, except for viscosity effects.

### VII. Diffusion

We are interested in the diffusion of a quantity that is concentrated in the vicinity of a point so that it converges towards zero within reasonable distance from that point, as described in Section 5.

We have to distinguish between the case that the space integral of the quantity is zero 7-1  $\int q dS = 0$

and when it is not. If the space integral is zero, the moment of inertia of the quantity is constant

$$7-2 \int r^2 q dS = \text{const}$$

This follows from applying the diffusion equation

$$7-3 \frac{\partial}{\partial t} q = \nu \nabla \cdot \nabla q$$

to (7-2) and applying Green's theorem, two times in succession.

The surface integrals vanish in consequence of the assumptions made regarding the decline of the quantity with increasing distance,

Section 5. Hence we obtain (7-2). More general, the first moment computed with the  $(2n)$ th power of  $r$  not being zero will remain

constant. It is practically sufficient for obtaining (7-2), that (7-1) be very small, that is that the Reynolds number  $(L^2/\nu t)$

be large where  $L$  and  $t$  denote respectively the largest distance and the largest time span involved in the experiment.

We have to distinguish between the moment of inertia of the quantity being finite or zero. There exist type preserving distributions of finite magnitude of the moment of inertia. They are the fundamental or dominating term of a combination of type perserving functions. They are the ones having a finite moment of inertia. The other ones have the moment of inertia zero.

These distributions, type-perserving under the action of viscosity only, have already been introduced into the theory in Ref. (k). However, they were only introduced for small Reynolds numbers. We see now that their field of usefulness is much broader. This statistically determined effect of viscosity brings this about. For the time being, and quite possibly for all time, it is necessary to restrict ourselves to using one prototype flow to serve once for all as correlation flow, and also by way of its inner part, as prototype lump flow. Going into the details of more than one lump flow seems to be quite at variance with the policy of a statistical approach. The prototype flow would take the place of the Hill vortex in Ref. (m).

The dominant influence of the viscosity demonstrated in Section 4 calls distinctly for the choice of a prototype flow self-preserving in type under viscosity effects, not under dynamic effect as the Hill vortex is. The first choice would naturally be the above fundamental self-preserving flow, having a finite moment of the velocity. This flow has also already appealed to the authors of Ref. (k) and (a) and has there, at least, been alluded to. This tends to show that

the supremacy of the turbulence was intuitively felt for many years. But it was not distinctly recognized nor broadly, confidently and cheerfully proposed.

We proceed to give some detail specification for this apparently popular and promising prototype flow.

viii. The prototype flow.

The flow is of axial symmetry, and is symmetric fore and aft. It can be described in a meridian plane using polar coordinates  $r$  and  $\varphi$ , the origin of the coordinate system coinciding with the lump center. The distance from the axis of symmetry is then given by  $r \sin \varphi$

The flow has the radial component of the velocity  $v$

$$7-1 \quad R = \cos \varphi e^{-r^2}$$

and a tangential velocity component at right angles to the radius

$$7-2 \quad T = -\sin \varphi (1-r^2) e^{-r^2}$$

The vorticity of the flow is in consequence

$$7-3 \quad e^{-r^2} r \sin \varphi (5-2r^2)$$

and thus its ratio to the axial distance  $r \sin \varphi$  is

$$7-4 \quad e^{-r^2} (5-2r^2)$$

This is constant over the surface of all concentric spheres  
 $r = \text{const.}$

The axial velocity component is zero at all points of a cylinder having unit distance from the axis of symmetry. All spheres about the origin are surfaces of normal velocity components proportional to  $\cos \varphi$ .

The flow shares this property with the surface of the Hill vortex.

The momentum integral of the correlation flow converges, and the total momentum is zero. The specification for this unit correlation flow describes only the type, but not the intensity. It does not indicate the specific kinetic energy of the turbulence flow. That depends on the intensity of the correlation flow, namely on the magnitude of its central velocity, at  $r = 0$ .

There are several reference lengths available for the correlation flow of the type (7-1, (7-2), the microscale  $\lambda$  of the unit flow, Ref. (t) is equal to  $\frac{1}{2}$ . The square of the radius of the sphere dividing the points of positive vorticity from those of negative vorticity is equal to 2.5. At the surface of that sphere, the vorticity is zero.

The outer portion of the correlation flow merely describes the statistical behavior of many adjacent lumps. The inner portions, the core, describes one average lump. The description is particularly immediate near the center. How far the territory of one lump extends depends on how many lumps there are per unit volume. That depends on the circumstances of the flow problem. Broadly spoken, the lumps have a tendency to swell out by viscosity effects until they come so close to each other that the mutual viscosity prevents further swelling out. The Reynolds number of the lump enters this picture. If the Reynolds number of the lumps becomes too large, each lump becomes a basic flow for smaller lumps. This points to an upper limit of the lump Reynolds number. All this will be discussed and computed, or at least estimated in a subsequent paper.

IX. Turbulence decay.

Decaying turbulence is by no means the principal problem of turbulence research. Quasi-steady turbulence is much more important. The decaying turbulence is however always prominently featured in the literature on the subject, because it is the only turbulence problem with respect to which some meager and incomplete theoretical results have been presented. Experimental information in that special field is also available.

Only two extreme cases are treated theoretically, the initial stage of turbulence decay and its final stage. The result regarding the initial stage is more acceptable than the other result.

The initial motion is considered to be a cluster of many smaller lumps, the cluster lump flow being the basic flow for the true lumps. The smaller lumps take the energy out of the cluster and dissipate it soon, so that the asymptotic history of the energy density can be obtained from the energy of the cluster or gross motion. The clusters are thinly distributed and therefore do not interfere with each other.

Under all these circumstances there is an analogy between the velocity of an ordinary lump and the energy of the cluster motion. Both are subject to ordinary diffusion. The shear mixing  $\mu$  is the analogue to the kinematic viscosity  $\nu$ . It happens that the final result is consistent with a constant mixing intensity, in that the Reynolds number of the small lumps does not change. The linear size increases proportional to the square root of the time, and the velocity, as we shall see, decreases inversely to the same.



While the space integral of the cluster energy  $\int E dS$  is not zero, the energy being a positive quantity, still the rate of decline of its moment of inertia is small. It is negligible in comparison with its own magnitude, because  $r^2 / (t \gamma)$  is large,  $r$  denoting the lump size and the  $t$  the time available. Under these circumstances, the moment of inertia of the energy does not change noticeably. Considering it to be dominantly contributed by a standard self-preserving diffusion distribution of finite moment of momentum of the energy, the energy at the cluster center would decline proportional to  $t^{-2.5}$ . But the cluster volume would increase proportional to  $t^{-1.5}$  so that the total energy per unit space would appear to be inverse to a linear function of the time. This is the result displayed in the literature and it seems to be confirmed experimentally. There are quite a number of assumptions or limiting conditions involved, but they can all be reasonably justified.

The opposite case is considered the final stage of decay. The current theory considers that then viscosity is supreme by reason of the small Reynolds numbers prevailing. That is hardly any longer turbulence, but is of interest as the asymptotic end of turbulence. Theory considers that unlimited space is available, also unlimited time to spread randomness all through this infinite space. The unevenness of the fluid motion assumes bigger and bigger scale, without limit. There remain always clusters, bigger and bigger in that the biggest clusters are decaying at the slowest rate. In accordance with the reasoning presented in Section 7, the energy decay will therefore be finally inverse to the 2.5th power of the time.

Will it really? What happens finally depends entirely on the boundary conditions in their widest sense including space and time.

Practically there will always be an energy supply by way of some noise effect. The decay will then be finally zero. If on the other hand, the fluid is surrounded by rigid energy-proof walls, the decay will finally be exponential. The decay exponent will increase from its initial value -1 because the turbulence becomes more crowded. It is not unusual nor unreasonable to expect that -2.5 will be reached and even surpassed. The question is whether the transient exponent -2.5 is less transient than the adjacent exponents, so that the decay curve is in particularly intimate contact with a parabola corresponding to that exponent.

If there are good reasons to expect that, then the finding of the present paper would make the exponent -2.5 even more acceptable in that the requirement for a small Reynolds number would be less severe.

The necessary condition for a good degree of preservation of the exponent -2.5 is that there are closely packed lump clusters and that there is furthermore enough space and a supply of even wider spread velocity configurations or deviations to become the feeding cluster flows for the less spread-out ones.

The semi-permanency of the exponent -2.5 can not be decisively indicated by experimental evidence. The precision of the tests is not large enough. There is however some experimental evidence to show that the exponent associated with the weakest energy intensity measureable is not inconsistent with an exponent of decay  $m = -2.5$ .

X. The ratio of the shear intensities.

Any statistical approach to the turbulence problem must have as its immediate aim the theoretical determination of the intensities of turbulence mixing. This mixing is not the primary subject of the present paper. The discussion of the relative weight of the viscosity effects and of the inertia effects must precede such determination. The determination of the intensity of mixing is a subject which requires its own study and its own paper. However, to offer in this paper at least some advance in the direction of that determination, we will conclude it with the discussion of the ratio of the shear mixing to the mixing for energy diffusion. That these two are different from each other, and why, will appear clearly in the following deduction. It will also appear that the general form used by us in Ref. (m) is consistent with the one immediately to be derived for the two mixing intensities. The present paper justifies thus the reasoning of this author in his previous paper on the subject.

The shear mixing expresses the transport of flow-wise momentum by the lumps crossing the shear or gliding surfaces. If the average velocity of the turbulence flow is  $u$ , then the flux of the fluid crossing in one or the other directions is approximately  $u/3$ . The momentum it transports depends on the penetration depth. This is somewhat analogous to the mean free path of the kinetic gas theory, but not closely analogous. We discussed this already in Ref. (m) and see now clearer for what reason the penetration depth  $f$  does not dominantly depend on random effects but on viscosity effects.

The velocity of a lump decreases with time. The rate of decrease is at least proportional to  $t^{-1}$ , which is for lumps spaced far apart from each other. In a quasi-steady shear flow, the lumps are not spaced far apart, but on the contrary, they must be considered closely packed. In that case, as discussed in Section 7, the reference velocities decrease proportional to  $t^{-2.5}$ . This time,  $t$ , has to be counted from an initial time point, such that at the time of the crossing  $t = t_0 = \text{const } \lambda^2 / \nu$  where  $\lambda$  denotes any reference length, such as for instance the microscale.

Hence, the penetration depth  $f$  results proportional to

$$10-1 \quad f_s = \int_{t_0}^{\infty} v dt = u_0 t_0 \int_0^1 \left(\frac{t}{t_0}\right)^{-2.5} d\left(\frac{t}{t_0}\right) \\ = \frac{u_0 t_0}{3 \cdot 1.5 \nu} = \frac{\lambda^2 u_0}{4.5 \nu}$$

Hence the shear mixing is proportional to

$$10-2 \quad m_0 = \frac{1}{4.5} \frac{\lambda^2 u_0^2}{\nu}$$

This is exactly the form proposed in Ref. (m).

For the diffusion of the energy we obtain a factor different from this. As the lump penetrates into a layer of different energy level, it does not really transport its energy that far, because much of it has been dissipated on the way. Hence an applicable penetration depth will be obtained, not by integrating the velocity, but by integrating its product with the energy still carried, and dividing the outcome by the initial energy. Thus

10-3

$$f_d = \int_{t_0}^{\infty} v E dt = u_0 t_0 \int_0^1 \left(\frac{t}{t_0}\right)^{-1.5} d\left(\frac{t}{t_0}\right) \quad .22.$$

$$= \frac{u_0 t_0}{3 \cdot 6.5} \gamma = \frac{\lambda^2 u_0}{19.5} \gamma$$

Hence it is demonstrated that the diffusion mixing is much smaller than the shear mixing. The ratio is about 3/13.

The smaller diffusion mixing would leave the influence of the secondary augmentation pronounced. However, the present derivation is based on the assumption that the lumps move straight ahead on the average. We have to examine how far that assumption is justifiable in shear flow.

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